Inverse and simulation filtering of digital seismograms

Inverse and simulation filtering of digital seismograms

NEXT: From filter problem to the simulation problem - the conversion of digital (broad-band) records into those from different seismograph systems.

Inverse and simulation filtering of digital seismograms

NEXT: From filter problem to the simulation problem - the conversion of digital (broad-band) records into those from different seismograph systems.

REASON: Signal amplitudes or onset time determination in a manner consistent with other observatories. Simulated systems will most commonly belong to the standard classes of instruments described by Willmore (1979) because there is no single, optimum class of instruments for the detection and analysis of different types of seismic waves.

Instrument classes

- High frequency teleseismic body waves: SP-instruments (class A)
- LP body waves and teleseismic surface waves: LP-instruments (class B)
- Regional body and surface waves: intermediate band (class C).
- Local magnitude: Wood-Anderson instrument.

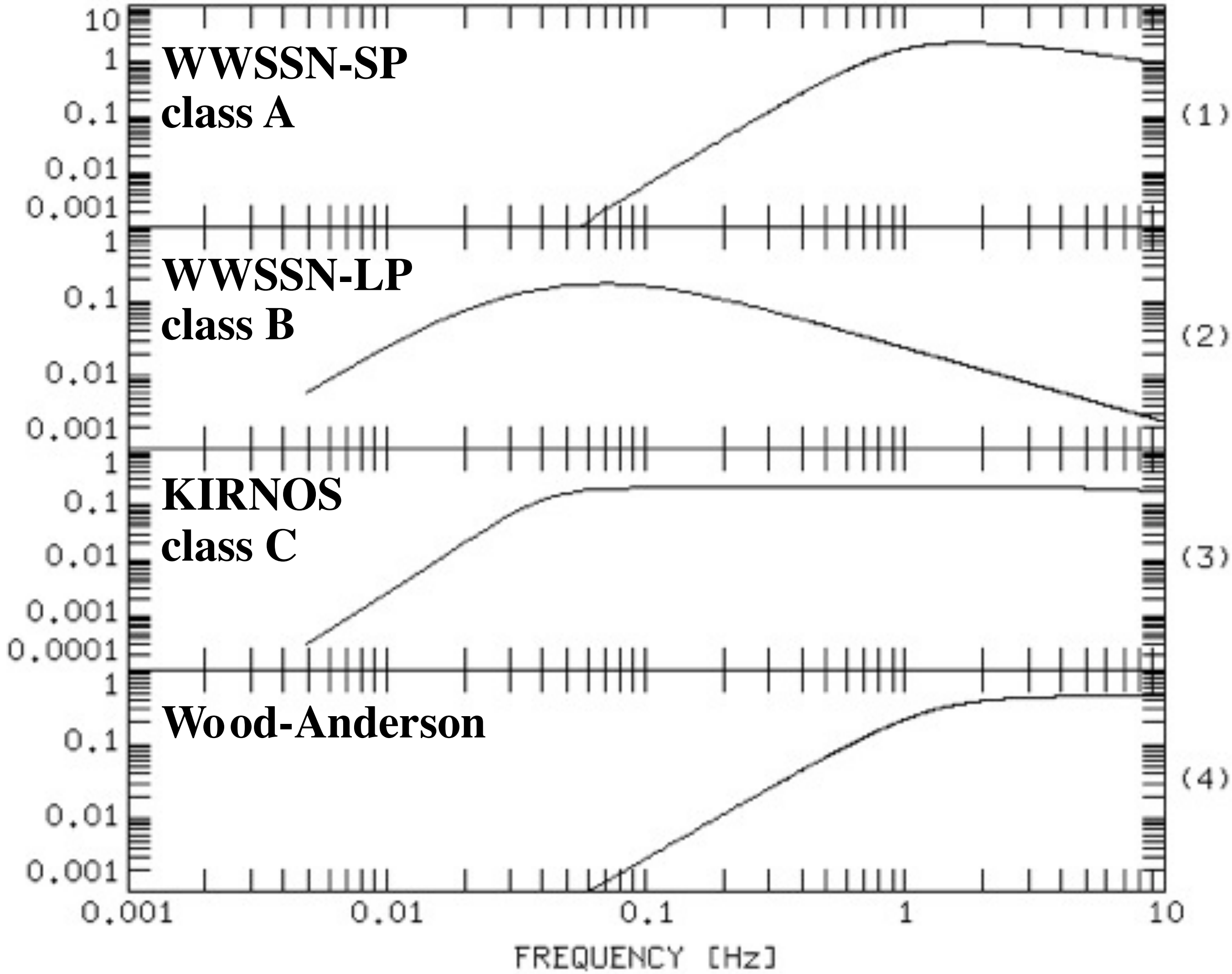


Fig. 9.1 Shapes of the moduli of the displacement frequency response functions of commonly used standard seismograph systems.

Simulation = Deconvolution + Filtering

$$Y_{sim}(z) = \frac{T_{syn}(z)}{T_{act}(z)} \cdot Y_{act}(z) = T_{sim}(z) \cdot Y_{act}(z)$$

$T_{act}(z)$ = transfer function of actual recording system

$T_{syn}(z)$ = transfer function of the instrument to be synthesized

$Y_{act}(z)$ = z- transform of the recorded seismogram

$Y_{sim}(z)$ = z- transform of the simulated seismogram

The concept of instrument simulation

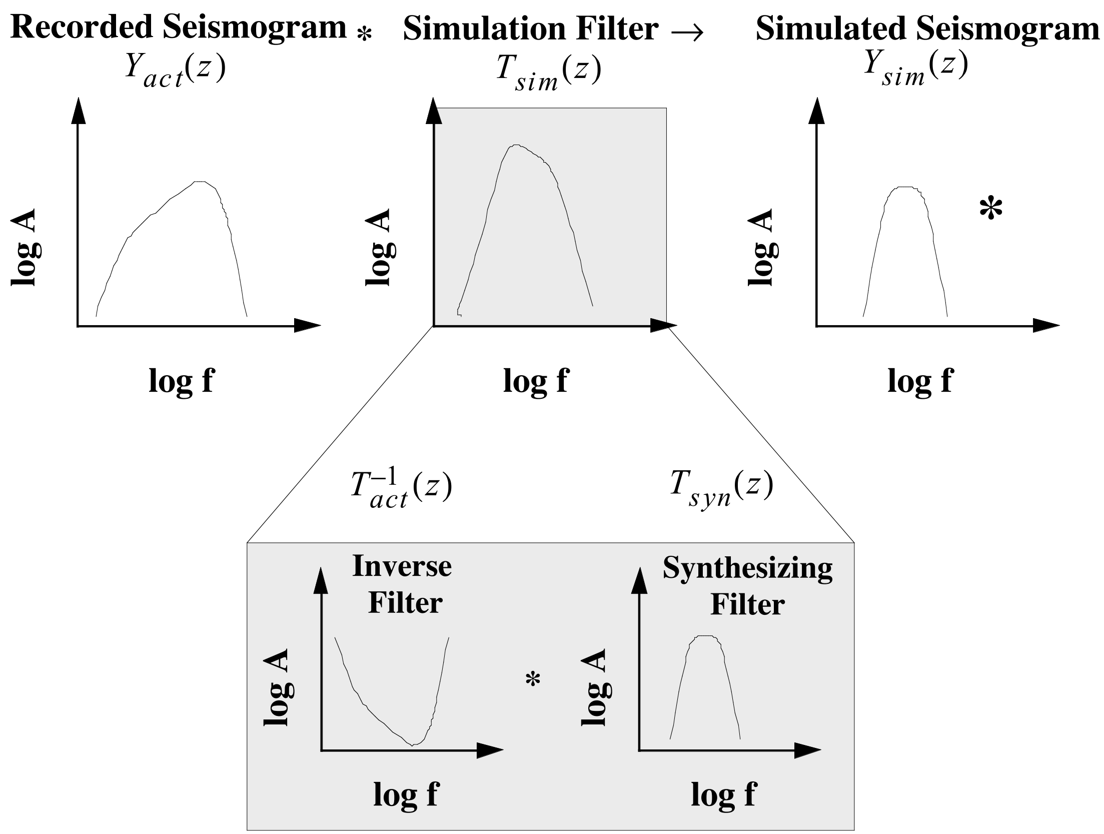


Fig. 9.2 The simulation of digital seismographs. The simulation filter can be thought of as a combination of an inverse filter for the actual recording system and a synthesizing filter for the simulated recording system. Displayed are schematic sketches of the amplitude frequency response functions of the contributing subsystems.

9.1 Stability problems

Noisefree situation

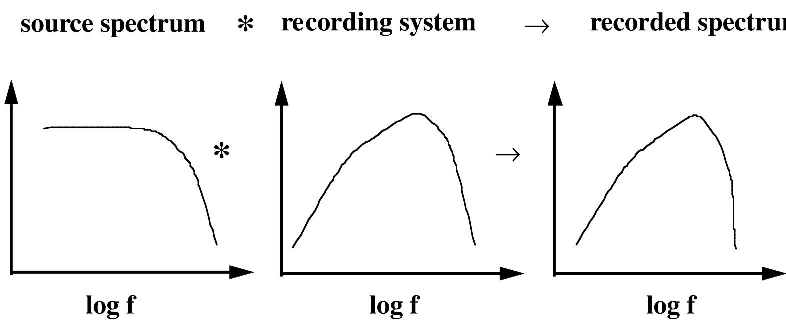


Fig. 9.3 Recording the displacement spectrum of an idealized earthquake source.

Recovery of source spectrum:

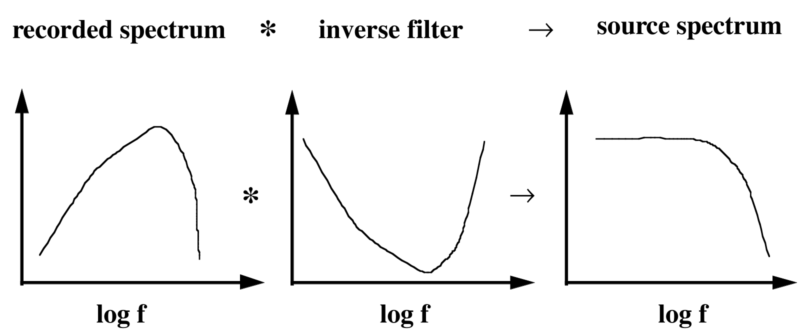


Fig. 9.4 Recovering the source spectrum by inverse filtering in the noisefree case.

Noisy situation

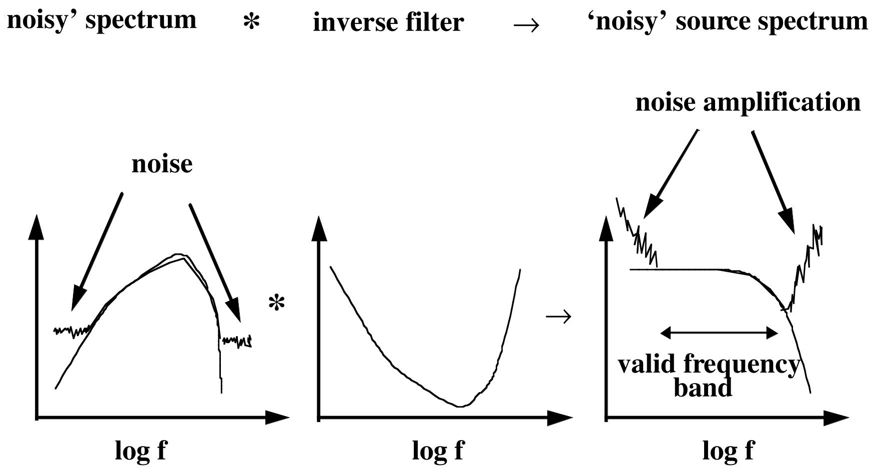


Fig. 9.5 Noise amplification by inverse filtering. The solid line in the left panel shows the signal plus noise while the noisefree signal is shown by the dashed line.

Noisy situation

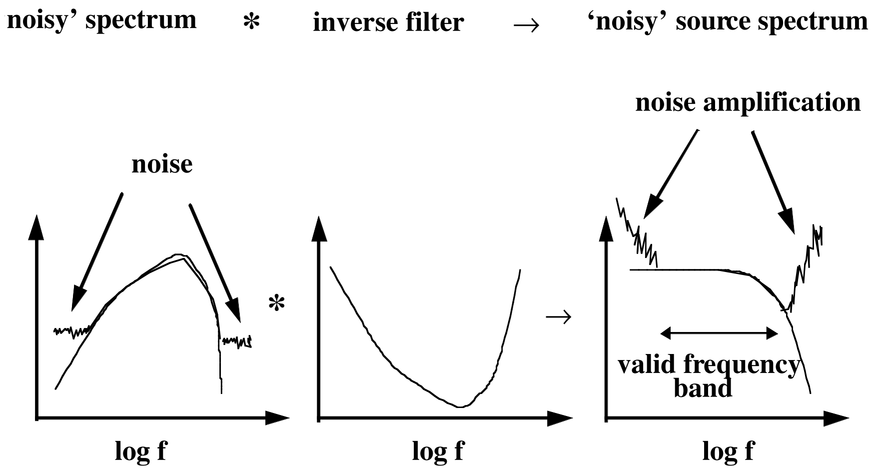


Fig. 9.5 Noise amplification by inverse filtering. The solid line in the left panel shows the signal plus noise while the noisefree signal is shown by the dashed line.

Problem:

- Decrease of signal-to-noise ratio (SNR) outside the pass-band of the recording instrument

- magnification of the inverse filter is largest where SNR is smallest. Thus, noise in this frequency band will be amplified (instability!).

Noisy situation

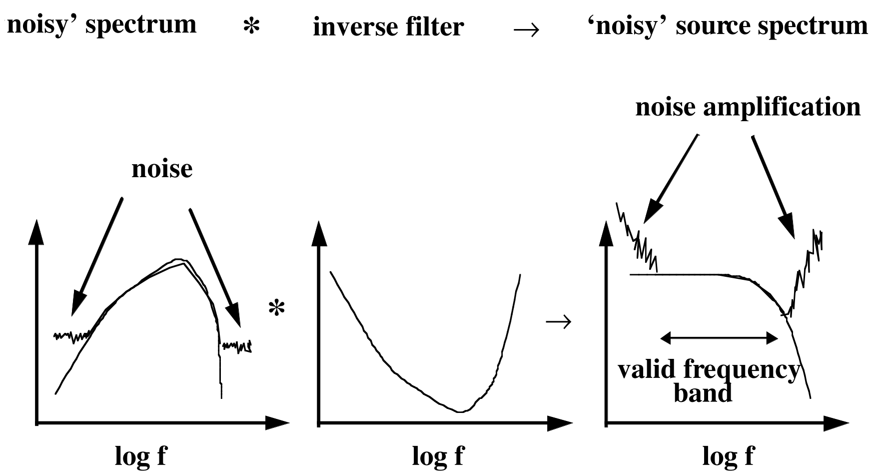


Fig. 9.5 Noise amplification by inverse filtering. The solid line in the left panel shows the signal plus noise while the noisefree signal is shown by the dashed line.

Problem:

- Decrease of signal-to-noise ratio (SNR) outside the pass-band of the recording instrument

- magnification of the inverse filter is largest where SNR is smallest. Thus, noise in this frequency band will be amplified (instability!).

Consequence:

- The instrument response can only be deconvolved within a certain *valid frequency band* in the presence of noise. The valid frequency band depends on both the signal-to noise ratio and the slope of the frequency response function of the recording systems.

Problem 3.8

From the shape of the frequency response function in the figure determine the poles and zeroes of the corresponding transfer function.

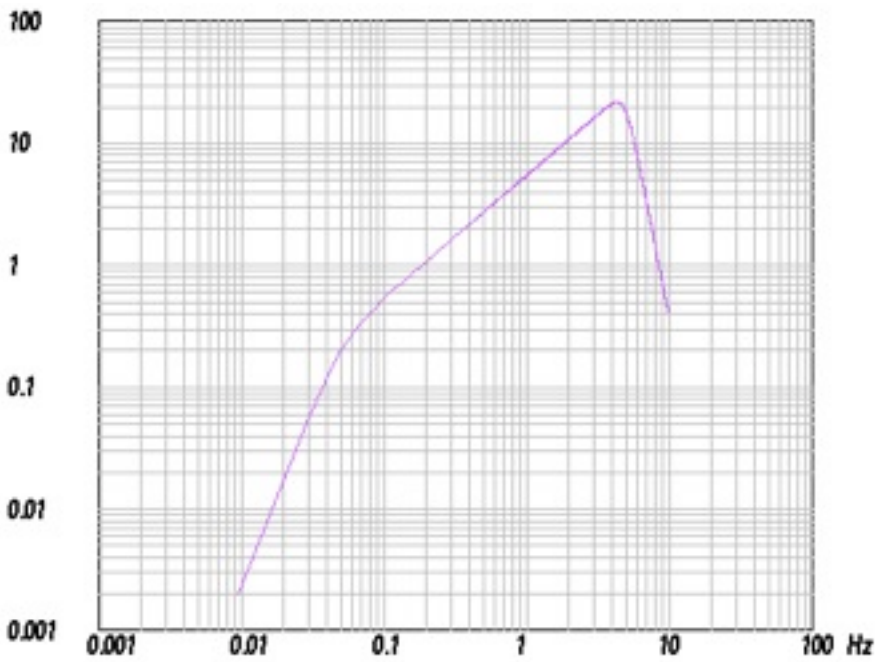


Fig. 3.4 Frequency response function (amplitude) with an unknown pole - zero distribution.

Solution 3.8 Within DST, the frequency response function of Problem 3.8 can be modeled by poles and zeros entered via the *Modify and Enter* option of the *Poles/ Zeros* menu. First select *problem3.8* from the *Load Response to Fit* option of the *Poles/Zeros* menu to display the frequency response function shown in Fig. 3.4. Next, estimate the different slopes and determine the number of poles and zeros which are needed to model them. One reasonable interpretation is sketched in Fig. A 3.15. Next, try to find the corner frequencies at which the changes in slope occur (here 0.05 Hz and 5 Hz).

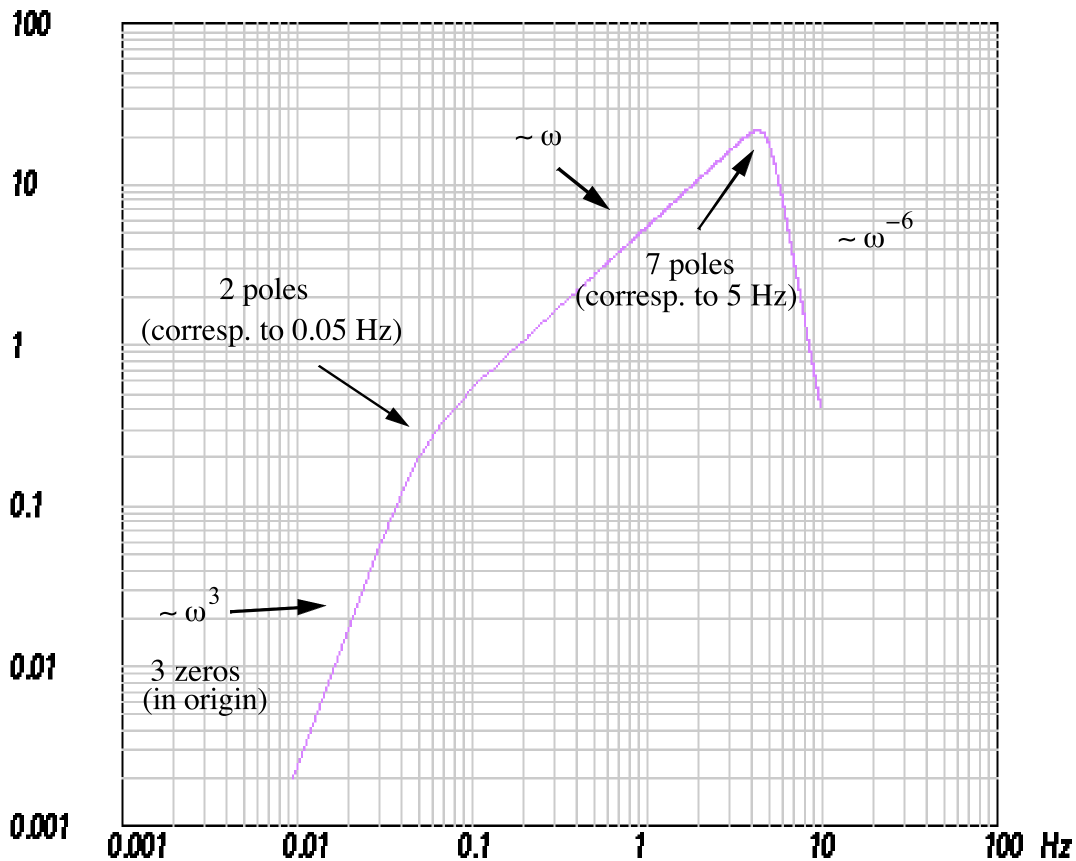


Fig. A 3.15 Frequency response function (amplitude) with an ’unknown’ pole - zero distribution from Problem 3.8.

(-0.2221, 0.2221)
(-0.2221, -0.2221)
(-28.473, 13.277)
(-28.473, -13.277)
(-20.194, 24.066)
(-20.194, -24.066)
(-8.131, 30.346)
(-8.131, -30.346)
(-31.416, 0)

This is displayed below (Fig. A 3.16).

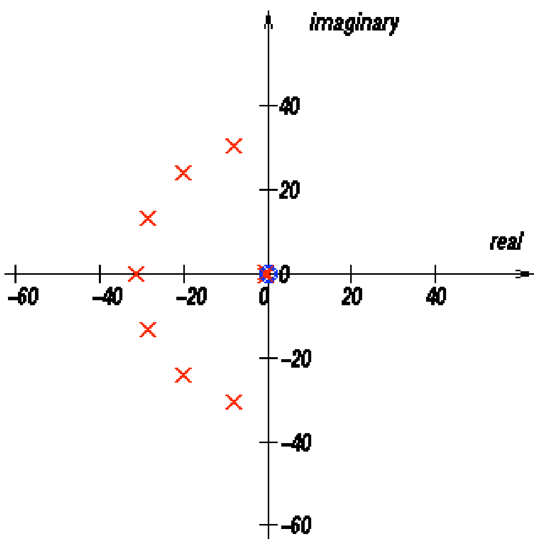


Fig. A 3.16 Trial pole/zero position to match the amplitude frequency response function of Problem 3.8.

At the corner frequency of 5 Hz, the frequency response function of Problem 3.8 has slightly higher amplitudes and shows a sharper change of slopes with respect to the trial frequency response function. This difference can be reduced by shifting some of the poles of the trial response closer to the imaginary axis to obtain a more "resonant" behaviour at that frequency.

The real distribution of poles and zeros for the frequency response function of Problem 3.8 is given below. It describes the frequency response function of the GRF array in SE Germany.

Poles:
(-0.2221, 0.2221)
(-0.2221, -0.2221)
(-7.0058, 30.6248)
(-7.0058, -30.6248)
(-19.5721, 24.5742)
(-19.5721, -24.5742)
(-28.3058, 13.6288)
(-28.3058, -13.6288)
(-31.4159, 0.0)

Zeros:
(0.0, 0.0)
(0.0, 0.0)
(0.0, 0.0)

Scale factor:
2.49059e10

For the generation of Fig. A 3.15 - Fig. A 3.17 within DST, an internal sampling frequency of 20 Hz and a window length of 2048 points was used.

Sampling and A/D conversion

Sampling and A/D conversion

= 2 step procedure

Sampling and A/D conversion

= 2 step procedure

• *Sampling or discretization* — Taking discrete samples of a continuous data stream. The data may still be in analog representation after the sampling process.

Sampling and A/D conversion

= 2 step procedure

• *Sampling or discretization* — Taking discrete samples of a continuous data stream. The data may still be in analog representation after the sampling process.

• *Analog to digital conversion (quantization and coding)* — For voltage signals, this steps normally occurs in an electronic device which is called ADC, 'analog to digital converter'.

The sampling process

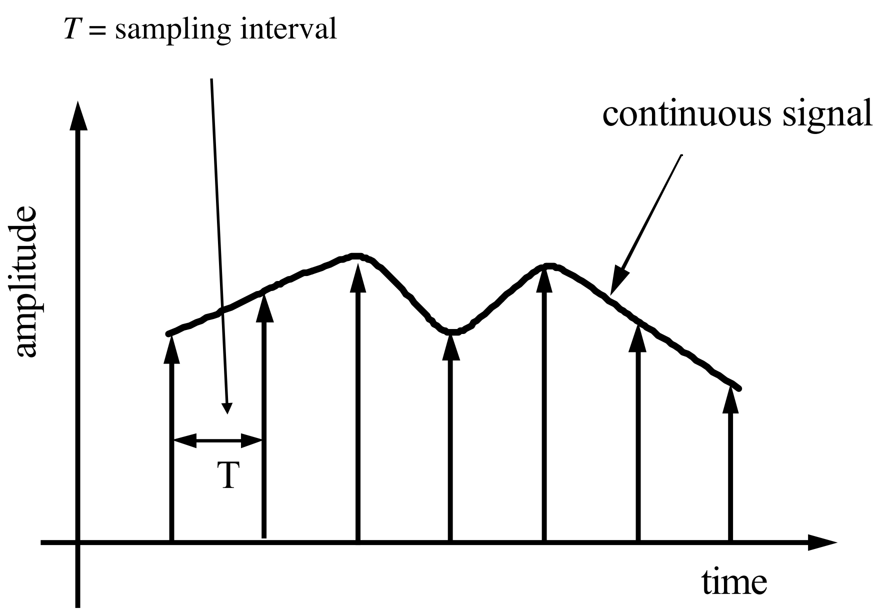


Fig. 5.1 Sketch of the discretization (sampling) process. The vertical arrows show the locations and the values of the samples. *T* denotes the sampling interval.

The sampling process

$1/T = f_{dig}$, is called the *sampling frequency* or the *digitization frequency*.

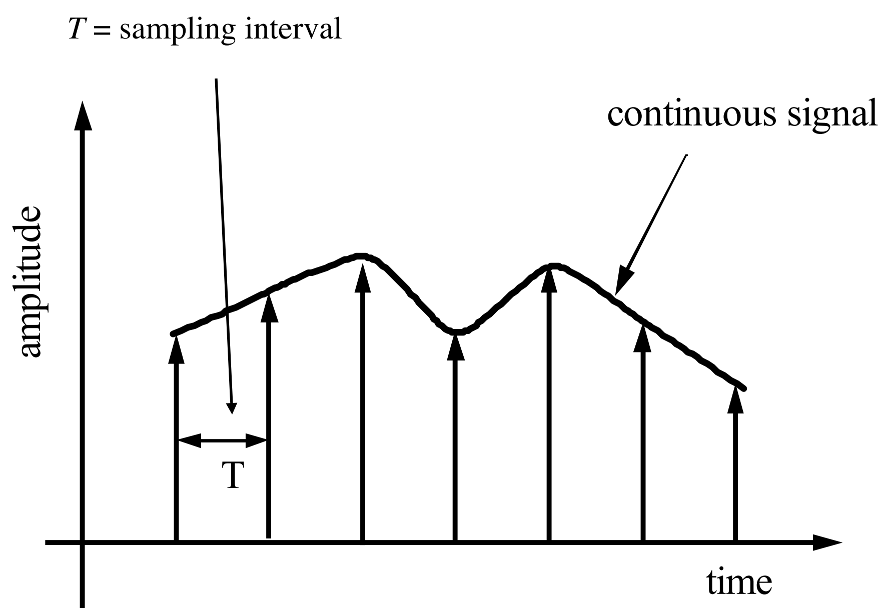


Fig. 5.1 Sketch of the discretization (sampling) process. The vertical arrows show the locations and the values of the samples. *T* denotes the sampling interval.

I) Discrete Signals

1.Discretization - Sampling

A continuous Signal Function: $x_a(t)$ taken at specific time steps T_s results in:

$$x[n] = x_a(nT_s);$$

T_s = sampling interval; $f_s = \frac{1}{T_s}$ = sampling rate or sampling frequency

Note! The amplitude values are still $x_a(t) \in \boldsymbol{R}$!!!!

The Sampling Theorem

In order to describe a continuous signal or function complete and unique using amplitude values taken at discrete times T_s, the sampled signal **MUST NOT HAVE** energy above a certain frequency $\frac{f_s}{2} = \frac{1}{2T_s}$. This frequency is also called **Nyquist-Frequency.**

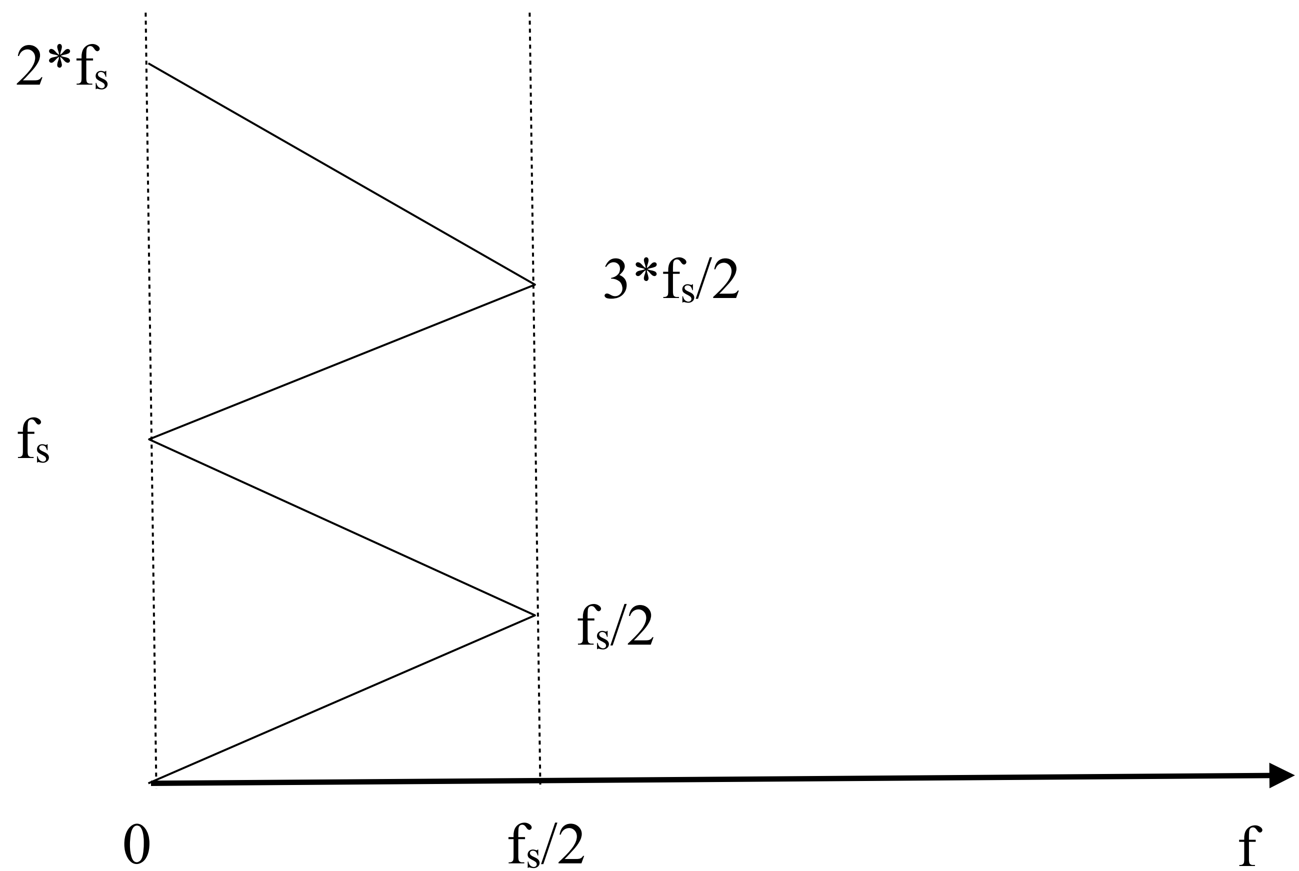
The corresponding continuous signal $x_a(t)$ could be reconstructed using a linear combination of the discrete function weighted by a function $\mathrm{sinc}(t) = \frac{\sin(t)}{t}$:

$$x_a(t) = \sum_{n=-\infty}^{\infty} x[n]\,\mathrm{sinc}(\pi f_s(t - nT_s))$$

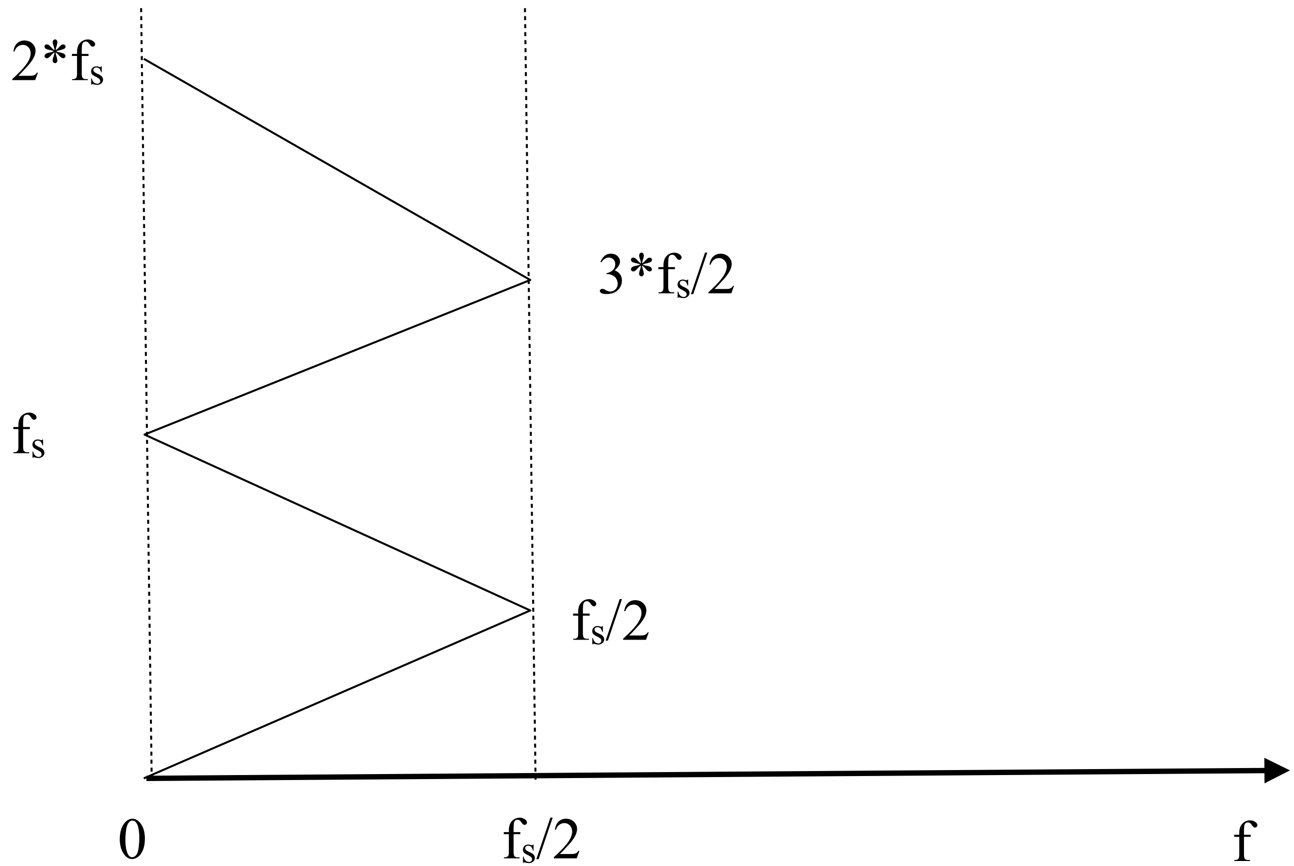
The sampling theorem **MUST** be applied **BEFORE** the sampling process. Therefore an analog lowpass filter must be applied before sampling - regardless which sampling frequency is used. The corner frequency (!) of that filter should satisfy:

$$f_c = 0.4 \cdot f_s.$$

Consequence of violation: **ALIASING**



Consequence of violation: **ALIASING**



Problem: We are sampling a continuous process with the sampling rate of 125 Hz. Estimate the alias frequencies of signals at 70, 120 and 300 Hz

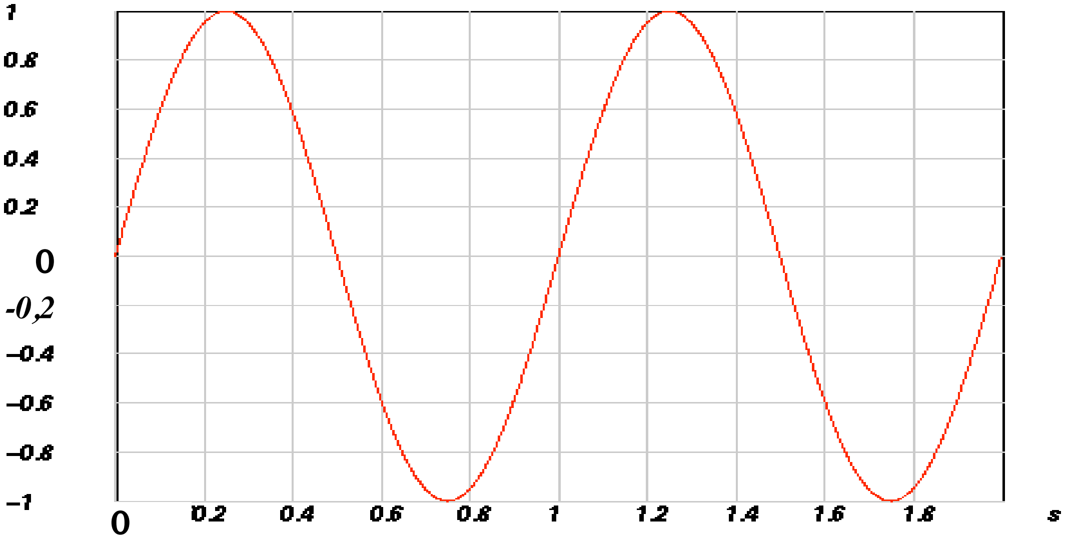


Fig. 5.2 Input signal for the simulation of the discretization process. The signal frequency is 1 Hz.

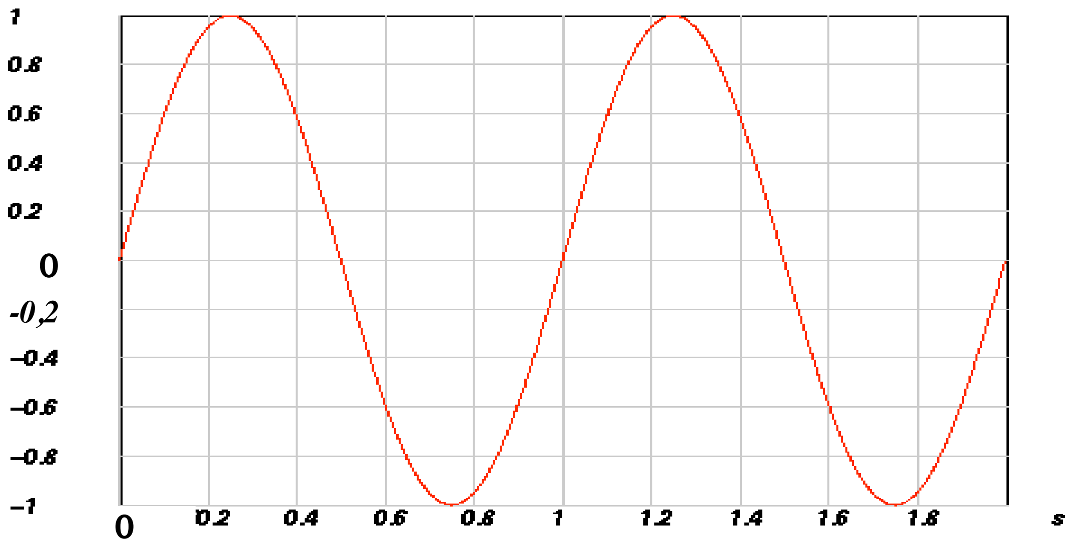


Fig. 5.2 Input signal for the simulation of the discretization process. The signal frequency is 1 Hz.

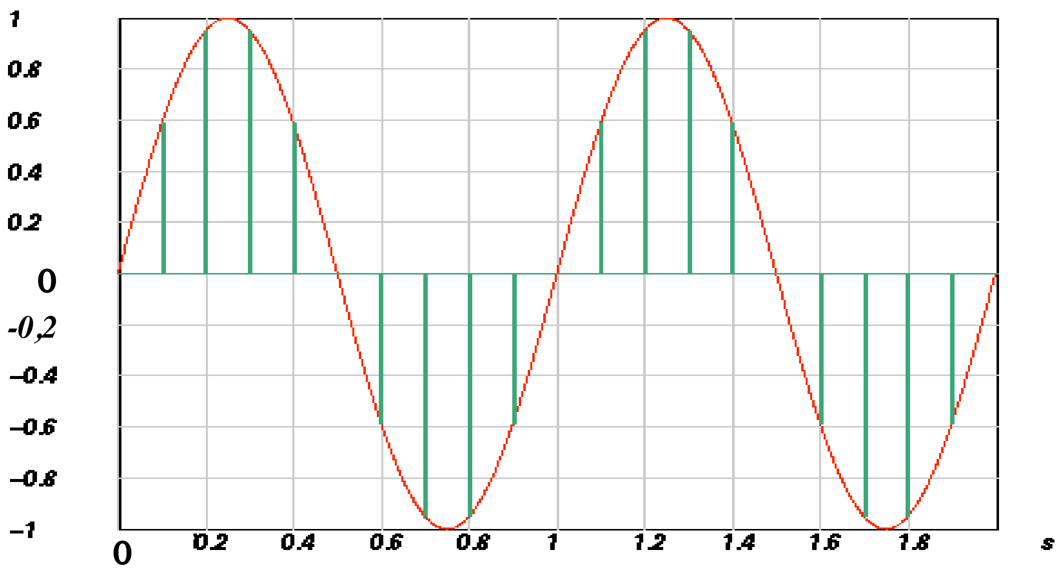


Fig. 5.3 Discretizing the data trace of Fig. 5.2 using a discretization frequency of 10 Hz. The vertical bars show the locations and the values of the function at the sampled times.

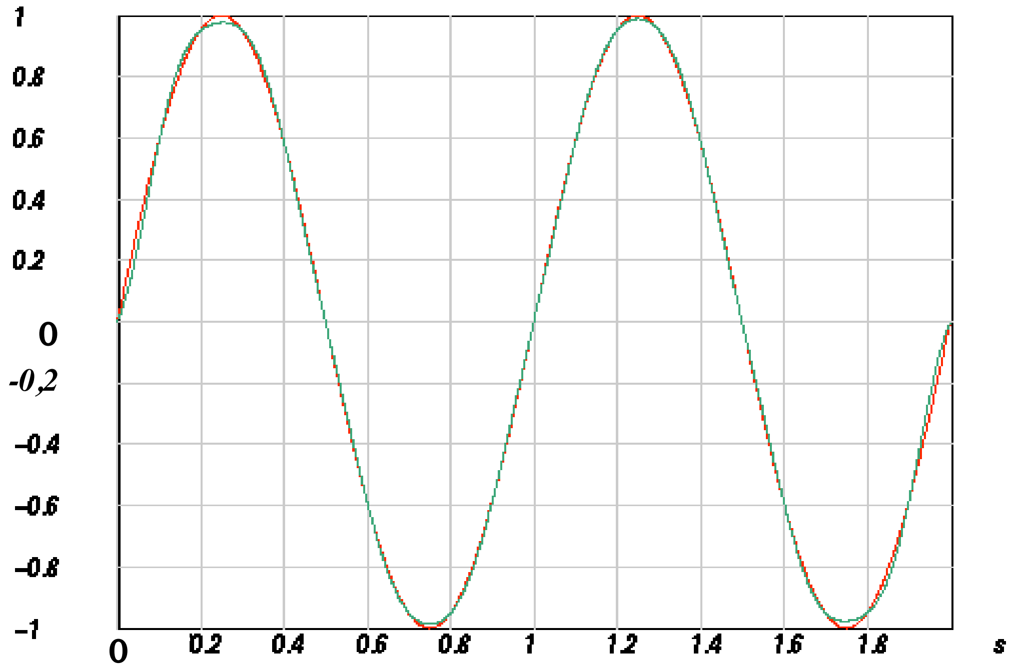


Fig. 5.4 Original and reconstructed trace of Fig. 5.2 (after discretizing all of them with 10 Hz prior to reconstruction).

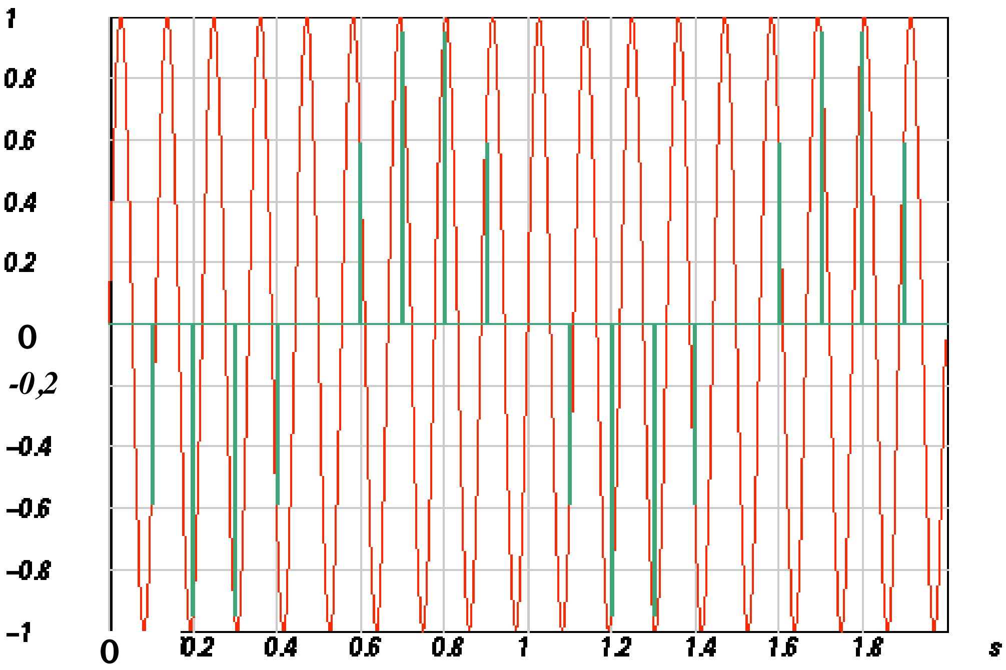


Fig. 5.6 Discretizing a sinusoidal signal with a signal frequency of 9 Hz and discretization frequency of 10 Hz. The vertical bars show the locations and the values of the function at the sampled times

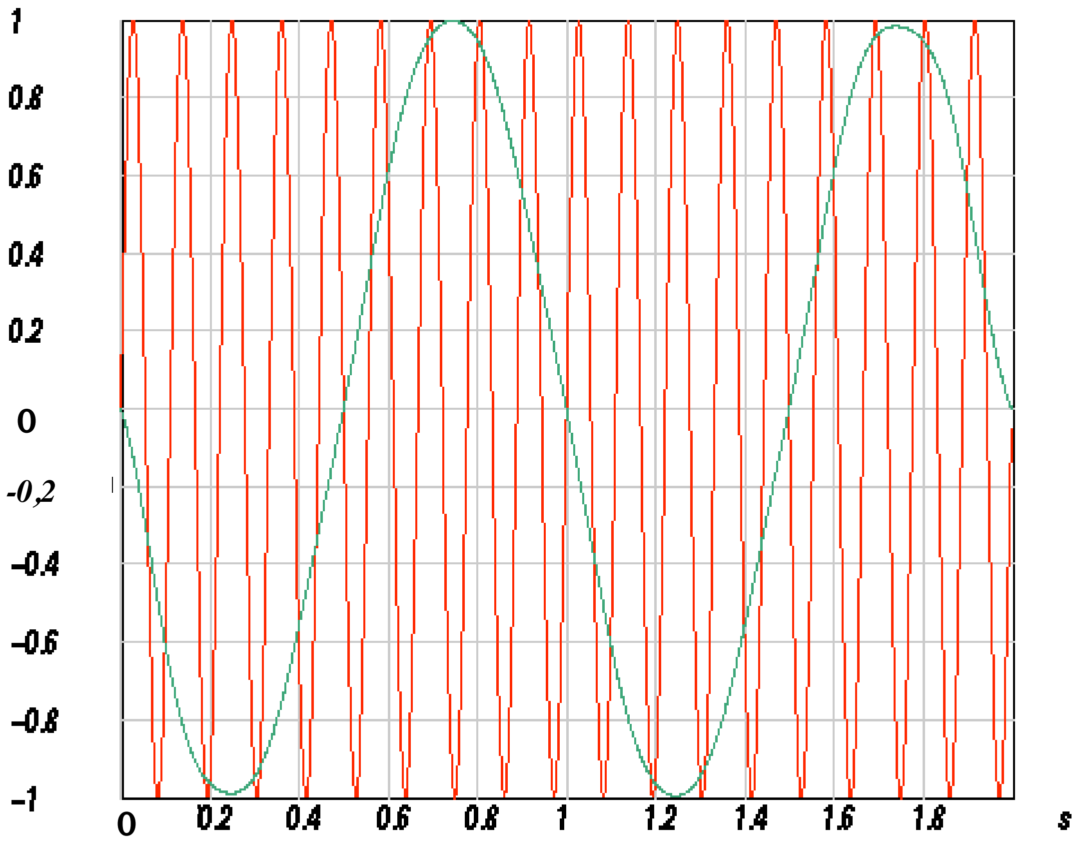


Fig. 5.5 Original and reconstructed sinusoidal signal with a signal frequency of 9 Hz (discretization frequency 10 Hz).

3.A/D-Conversion

Decimal system:

$$x_{(10)} = \sum_i d_i^{(10)} 10^i;$$

Example:

$$1024_{(10)} = 4 \cdot 10^0 + 2 \cdot 10^1 + 0 \cdot 10^2 + 1 \cdot 10^3$$

LSB MSB

Binary system:

$$x_{(2)} = \sum d_i^{(2)} 2^i$$

Example:

$512_{(10)} = 0 \cdot 2^0 + \ldots + 0 \cdot 2^8 + 1 \cdot 2^9$ represents “Little Endian”

LSB MSB 000000001

A 16 bit A/D-converter could represent in principle 2^{16} output states in its maximum (values between 0 -(2^n-1) are possible).

The LSB (least significant bit) or smallest step width of the A/D-converter (resolution) is defined by:

$$LSB = \frac{\text{Maximale Voltage}}{2^n} = Q.$$

As the resolution is directly dependent on the number of bits, a n-bit A/D-converter has “n-bit” resolution. Unfortunately, there is no rule, which would specify a “critical” number of “must have” bits. It is simply like that: if we have more bits we will decrease the noise added to the signal

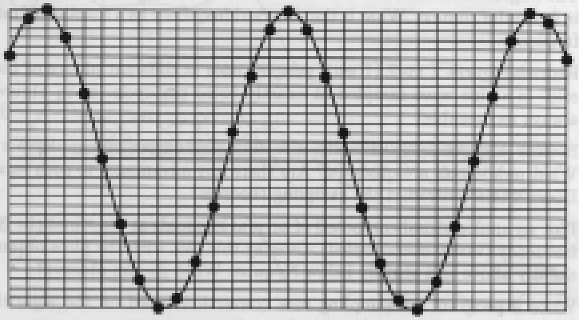


Figure 2.20: Conversion of an analog signal into a corresponding digital one involves quantizing both axes, sampling time and digitizing signal value. In the figure we see the original analog signal overlaid with the sampled time and digitized signal value grid. The resulting digital signal is depicted by the dots.

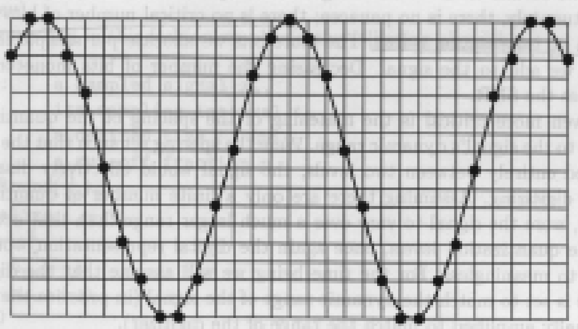


Figure 2.21: Conversion of an analog signal into the corresponding digital one with fewer digitizing levels. As in the previous figure the original analog signal has been overlaid with the sampled time and digitized signal value grid. However, here only 17 levels (about four bits) are used to represent the signal.

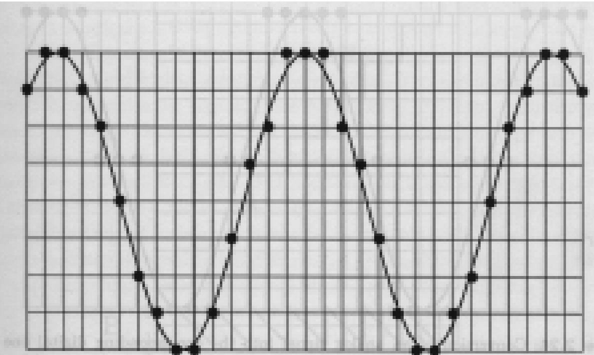


Figure 2.22: Conversion of an analog signal into the corresponding digital one with fewer digitizing levels. Once again the original analog signal has been overlaid with the sampled time and digitized signal value grid. Here only nine levels (a little more than three bits) are used to represent the signal.

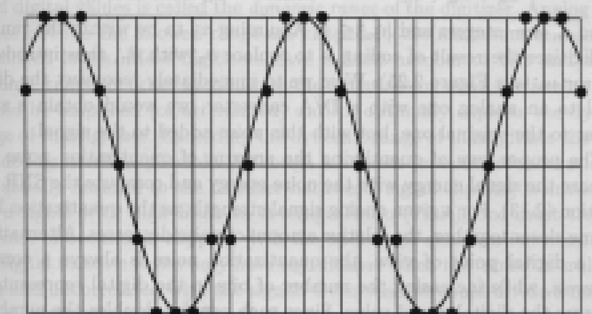


Figure 2.23: Conversion of an analog signal into the corresponding digital one with fewer digitizing levels. Once again the original analog signal has been overlaid with the sampled time and digitized signal value grid. Here only five levels (about two bits) are used to represent the signal.

An equivalent important parameter of A/D conversion is the so called dynamic range:

$$D = 20\log_{10}\left(\frac{A_{max}}{A_{min}}\right)$$

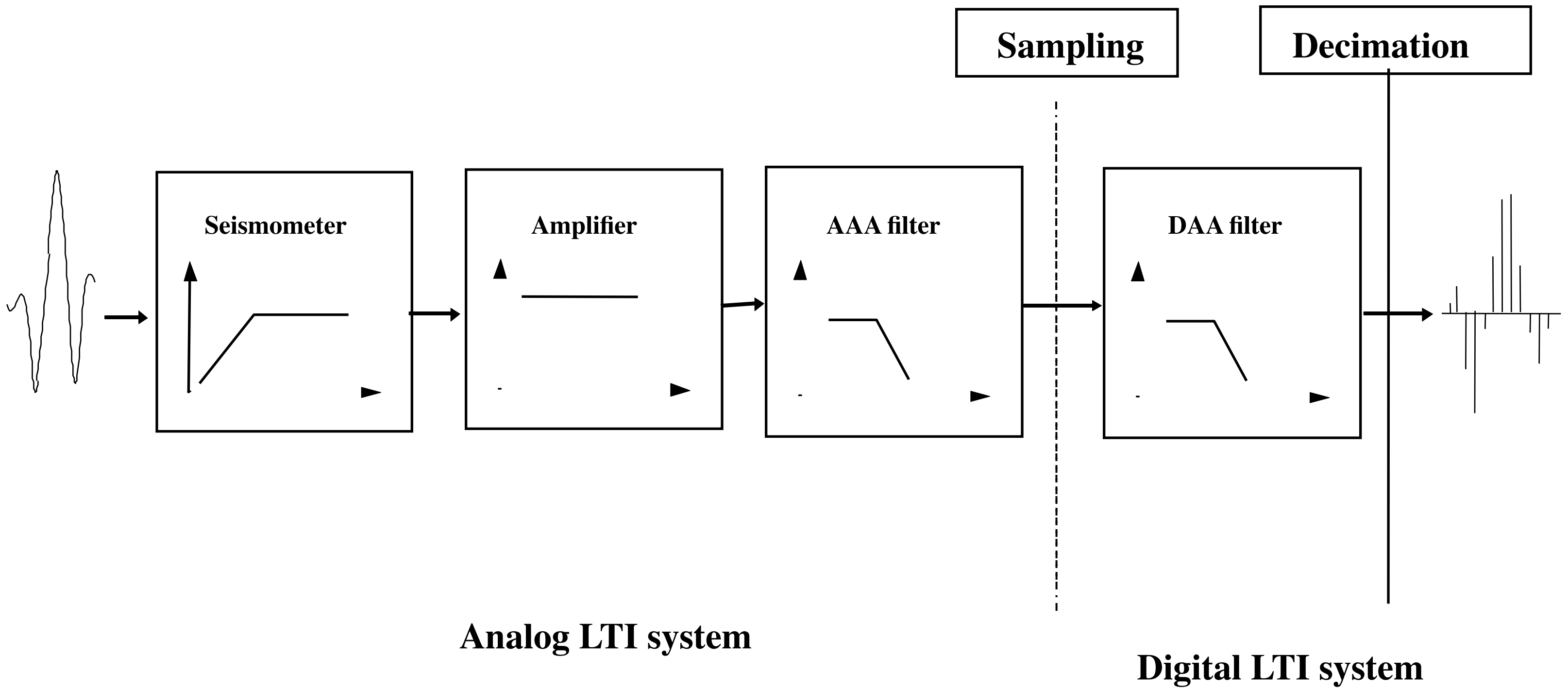
and therefore

$$D = 20\log_{10}(2^n - 1) \approx n\log_{10}(2) = n \cdot 6$$

16 bit A/D-converter: 90dB;
24 bit A/D-converter: 138 dB;

Be aware of the sign!

FIR - Filter Effects

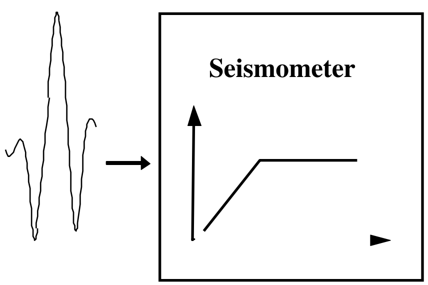


Why bothering?

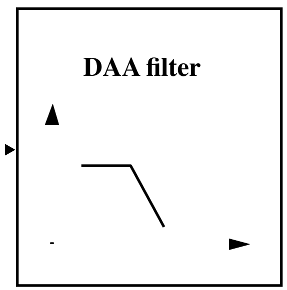
What is the reason for doing FIR filtering and decimating?

Nearly all seismic recorders use the oversampling technique to increase the resolution of recordings. In order to achieve an optimum valid frequency band, the filters are very steep.

Besides its advantages this also bears new problems.




```
#            << IRIS SEED Reader, Release 4.4 >>
#
#            ======== CHANNEL RESPONSE DATA ========
B050F03    Station:     RJOB
B050F16    Network:     BW
B052F03    Location:    ??
B052F04    Channel:     EHZ
B052F22    Start date:  2007,199
B052F23    End date:    No Ending Time
#            =======================================
#            +              +------------------------------------------+              +
#            +              |   Response (Poles & Zeros),  RJOB ch EHZ   |              +
#            +              +------------------------------------------+              +
#
B053F03    Transfer function type:             A [Laplace Transform (Rad/sec)]
B053F04    Stage sequence number:                1
B053F05    Response in units lookup:            M/S - Velocity in Meters per Second
B053F06    Response out units lookup:            V - Volts
B053F07    A0 normalization factor:             6.0077E+07
B053F08    Normalization frequency:              1
B053F09    Number of zeroes:                   2
B053F14    Number of poles:                    5
#            Complex zeroes:
#              i  real          imag          real_error    imag_error
B053F10-13    0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
B053F10-13    1  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
#            Complex poles:
#              i  real          imag          real_error    imag_error
B053F15-18    0 -3.700400E-02  3.701600E-02  0.000000E+00  0.000000E+00
B053F15-18    1 -3.700400E-02 -3.701600E-02  0.000000E+00  0.000000E+00
B053F15-18    2 -2.513300E+02  0.000000E+00  0.000000E+00  0.000000E+00
B053F15-18    3 -1.310400E+02 -4.672900E+02  0.000000E+00  0.000000E+00
B053F15-18    4 -1.310400E+02  4.672900E+02  0.000000E+00  0.000000E+00
```




```
#          +                  +--------------------------------+                  +
#          +                  |  FIR response,  RJOB ch EHZ  |                  +
#          +                  +--------------------------------+                  +
#
B061F03    Stage sequence number:               3
B061F05    Symmetry type:                   A
B061F06    Response in units lookup:            COUNTS - Digital Counts
B061F07    Response out units lookup:           COUNTS - Digital Counts
B061F08    Number of numerators:                96
#          Numerator coefficients:
#            i, coefficient
B061F09       0  3.767143E-09
B061F09       1  5.277283E-07
B061F09       2  2.184651E-06
B061F09       3 -5.639535E-06
B061F09       4 -1.233773E-06
B061F09       5  9.386712E-06
B061F09       6  4.859924E-06
B061F09       7 -1.644319E-05
…
#
#          +                  +------------------------------+                  +
#          +                  |  Decimation,  RJOB ch EHZ  |                  +
#          +                  +------------------------------+                  +
#
B057F03    Stage sequence number:               4
B057F04    Input sample rate:               1.000000E+03
B057F05    Decimation factor:               5
B057F06    Decimation offset:               0
B057F07    Estimated delay (seconds):           1.490000E-01
B057F08    Correction applied (seconds):        0.000000E+00
```

Linear Difference Equation

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{l=0}^{M} b_l x[n-l]$$

Infinite Impulse Response: $a_k \neq 0$

Finite Impulse Response: $a_0 = 1$; $a_{k\neq 0} = 0$

- FIR filters :

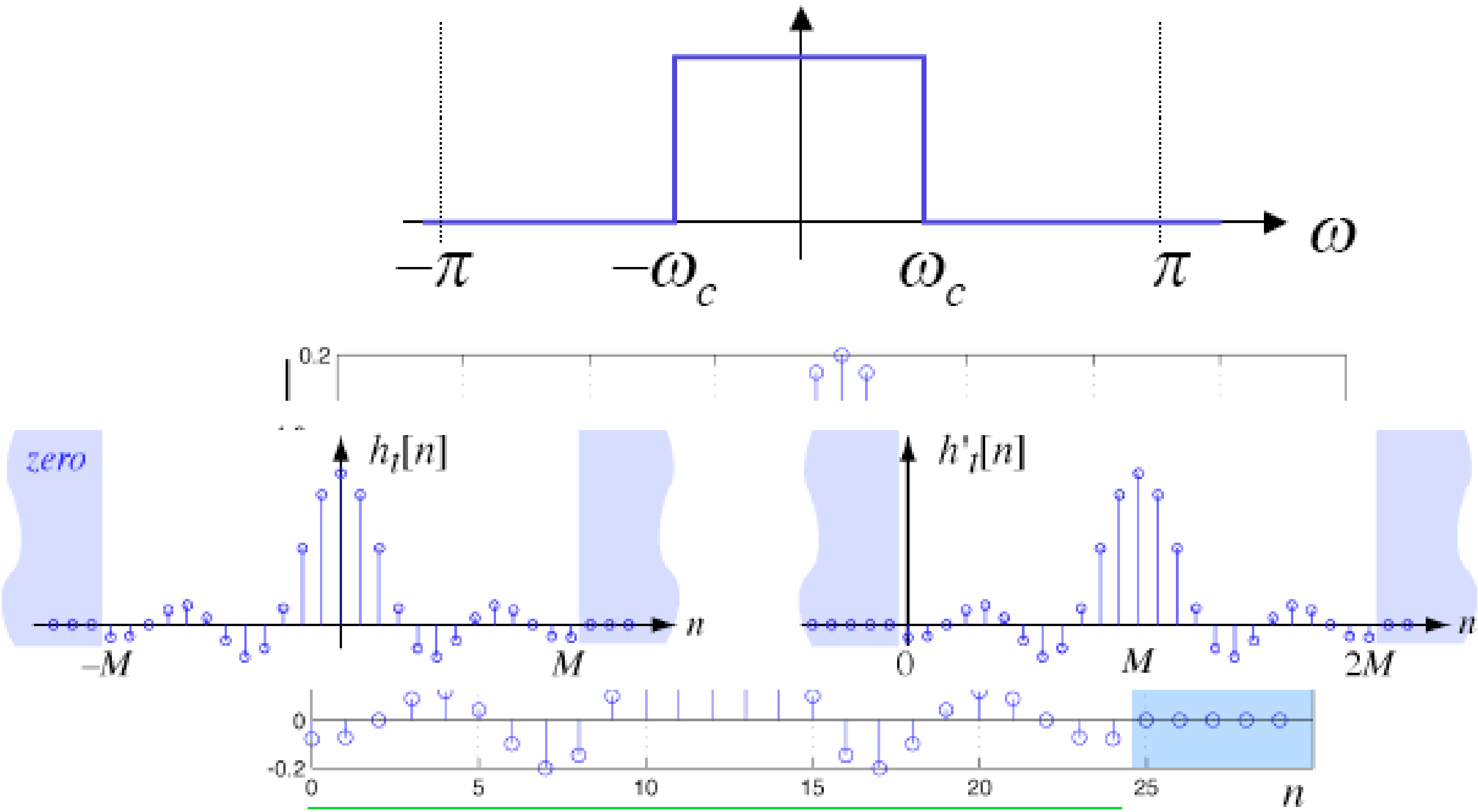
+ Always stable.
- Steep filters need many coefficients.
+ Both causal and noncausal filters can be implemented.
+ Filters with given specifications are easy to implement!

- IIR filters :

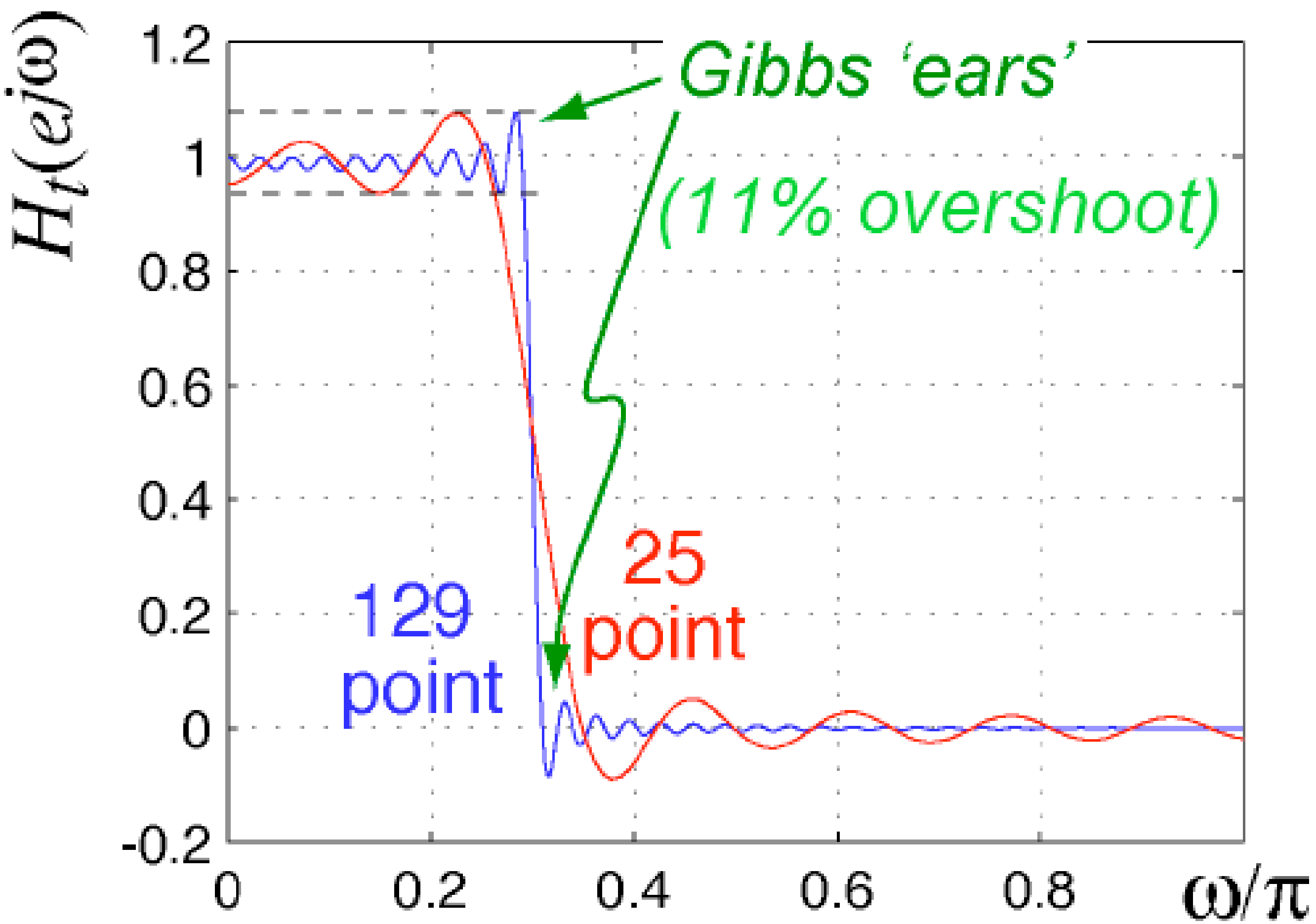
- Potentially unstable and subject to quantization errors.
+ Steep filters can easily be implemented with a few coefficients. Speed.
- Filters with given specifications are in general, difficult, if not impossible, to implement *exactly(!)*.

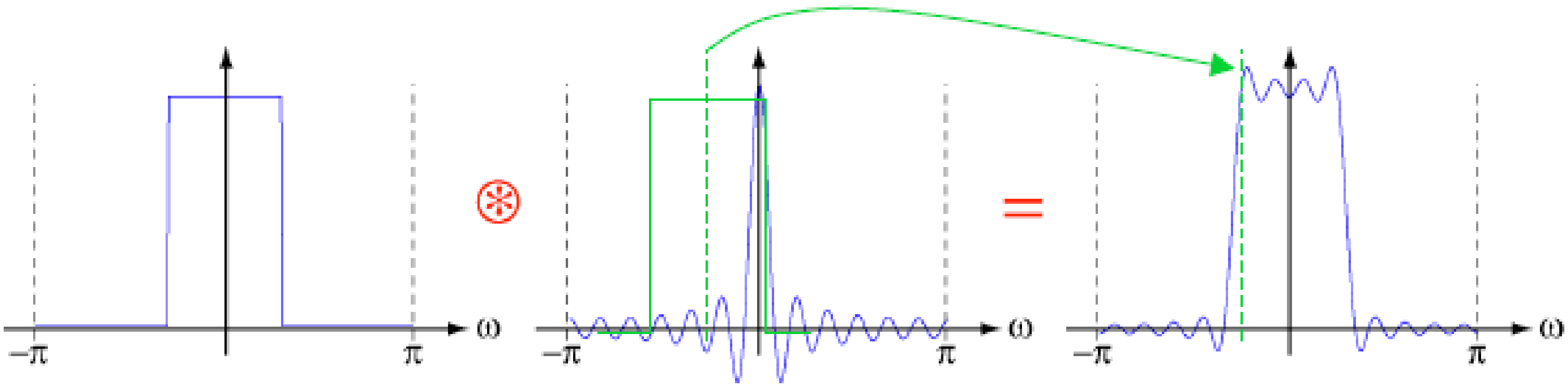
In SEED the impulse response of the decimation filters are given. But how to construct FIR filters?

Easiest way: inverse DFT with selected spectral shape and phase and truncate the (infinite) sequence to form a finite impulse response



Filter length vs. Steepnes





$H_d(e^{j\omega})$ $\quad$ $\mathrm{DTFT}\{w_R[n]\}$ $\quad$ $H_t(e^{j\omega})$

periodic sinc...

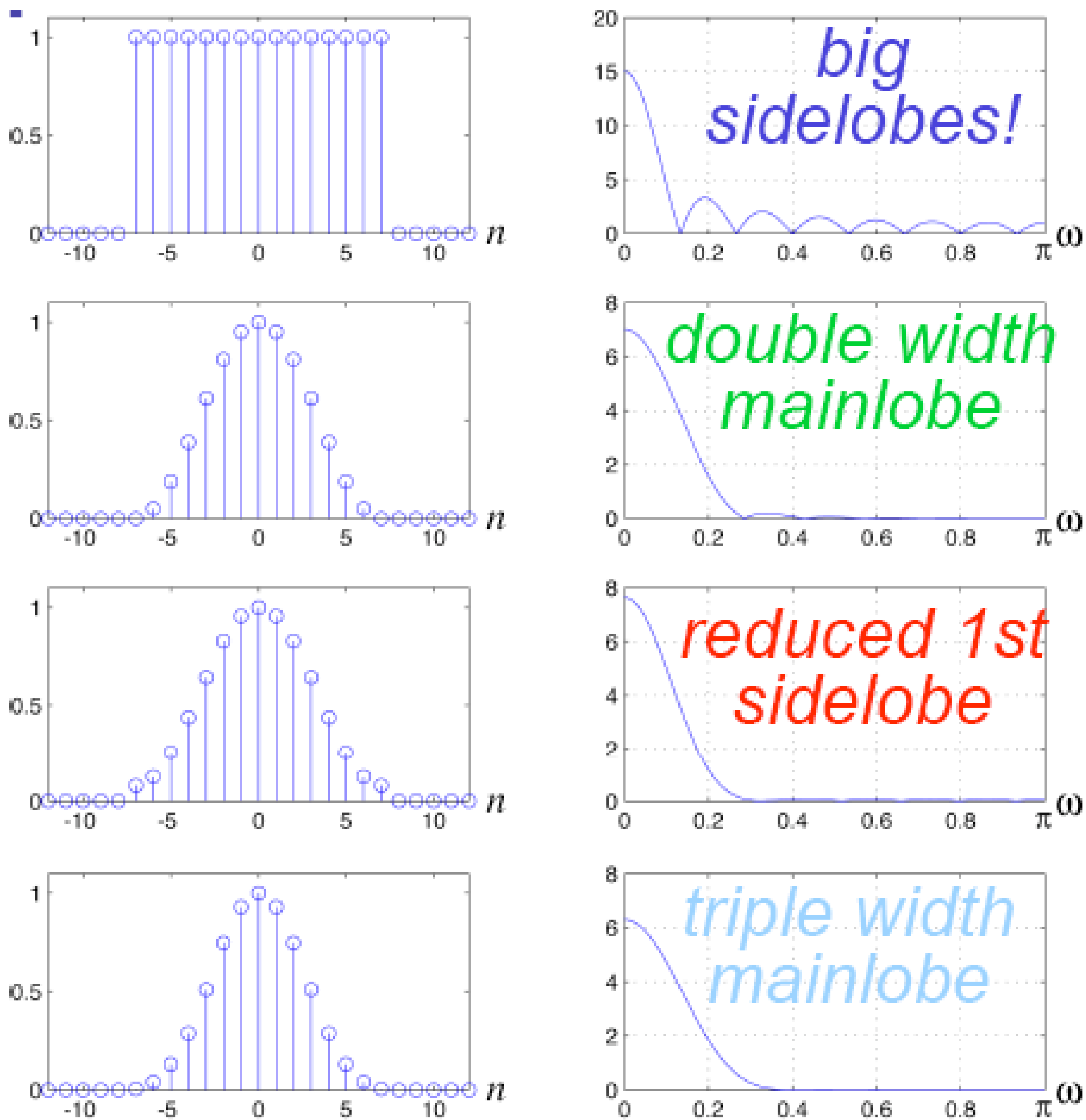
$$\frac{\sin Nx}{\sin x}$$

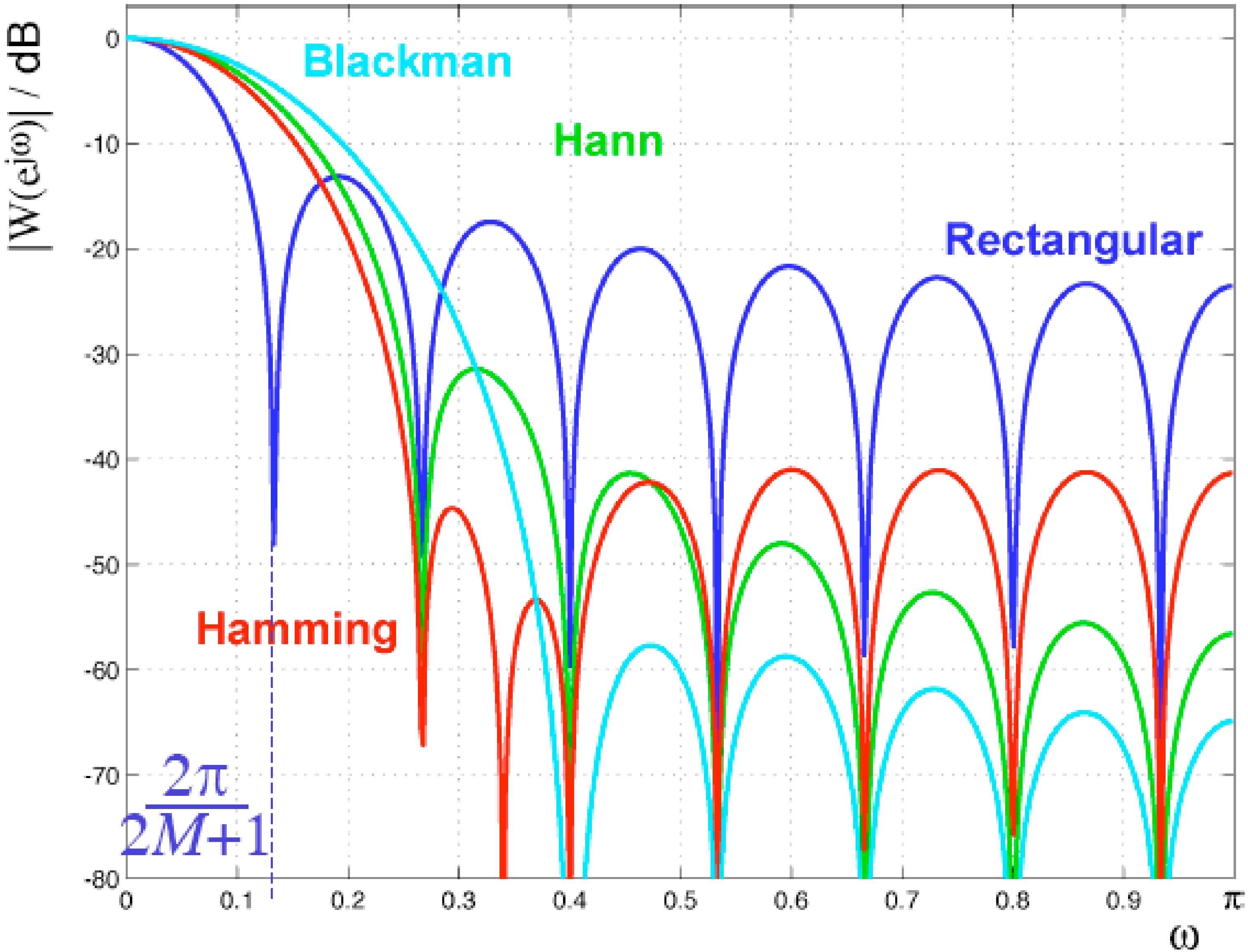
Rectangular

Hanning

Hamming

Blackman





Blackman
Hann
Rectangular
Hamming
|W(e^jω)| / dB
ω
$\frac{2\pi}{2M+1}$
0
-10
-20
-30
-40
-50
-60
-70
-80
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
π

Windowed FIR Filter Example:

1. Get ideal filter impulse response:

$$\omega_c = 0.15\pi \qquad \Rightarrow h_d[n] = \frac{\sin 0.15\pi n}{\pi n}$$

2. Get window function for truncation:

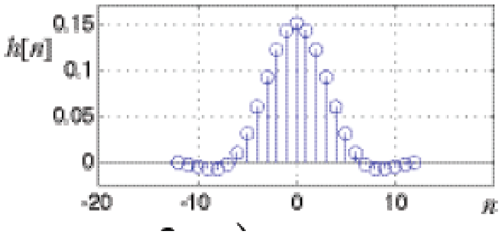
$$N = 25 \rightarrow M = 12 \quad (\text{N=2M+1})$$

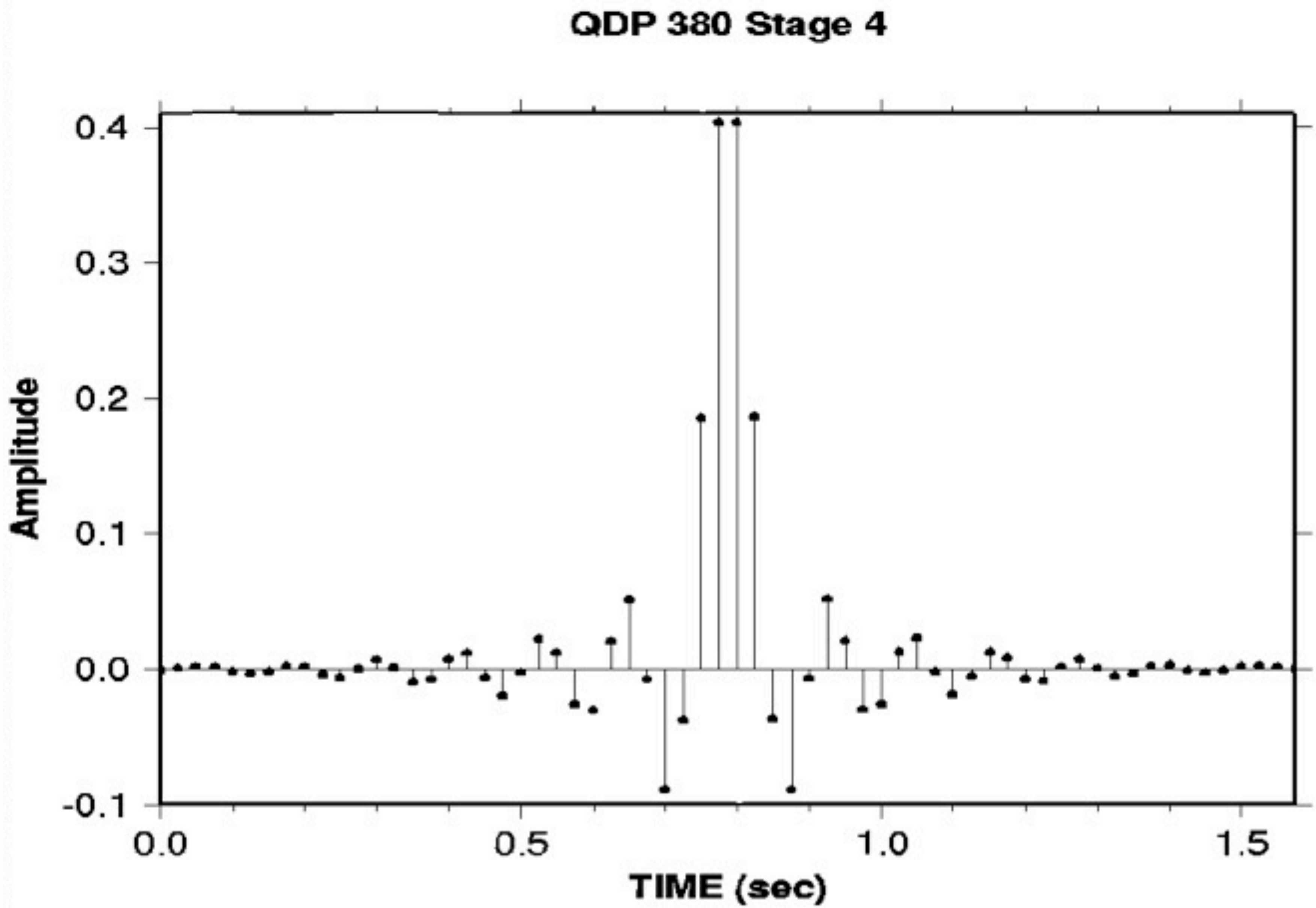
$$\Rightarrow w[n] = 0.54 + 0.46\cos\left(2\pi\frac{n}{25}\right) \quad -12 \le 12$$

3. Apply window:

$$h[n] = h_d[n]w[n]$$

$$= \frac{\sin 0.15\pi n}{\pi n}\left(0.54 + 0.46\cos\frac{2\pi n}{25}\right) \quad -12 \le 12$$




QDP 380 Stage 4
Amplitude
0.4
0.3
0.2
0.1
0.0
-0.1
0.0
0.5
1.0
1.5
TIME (sec)

Zero Phase FIR Filter

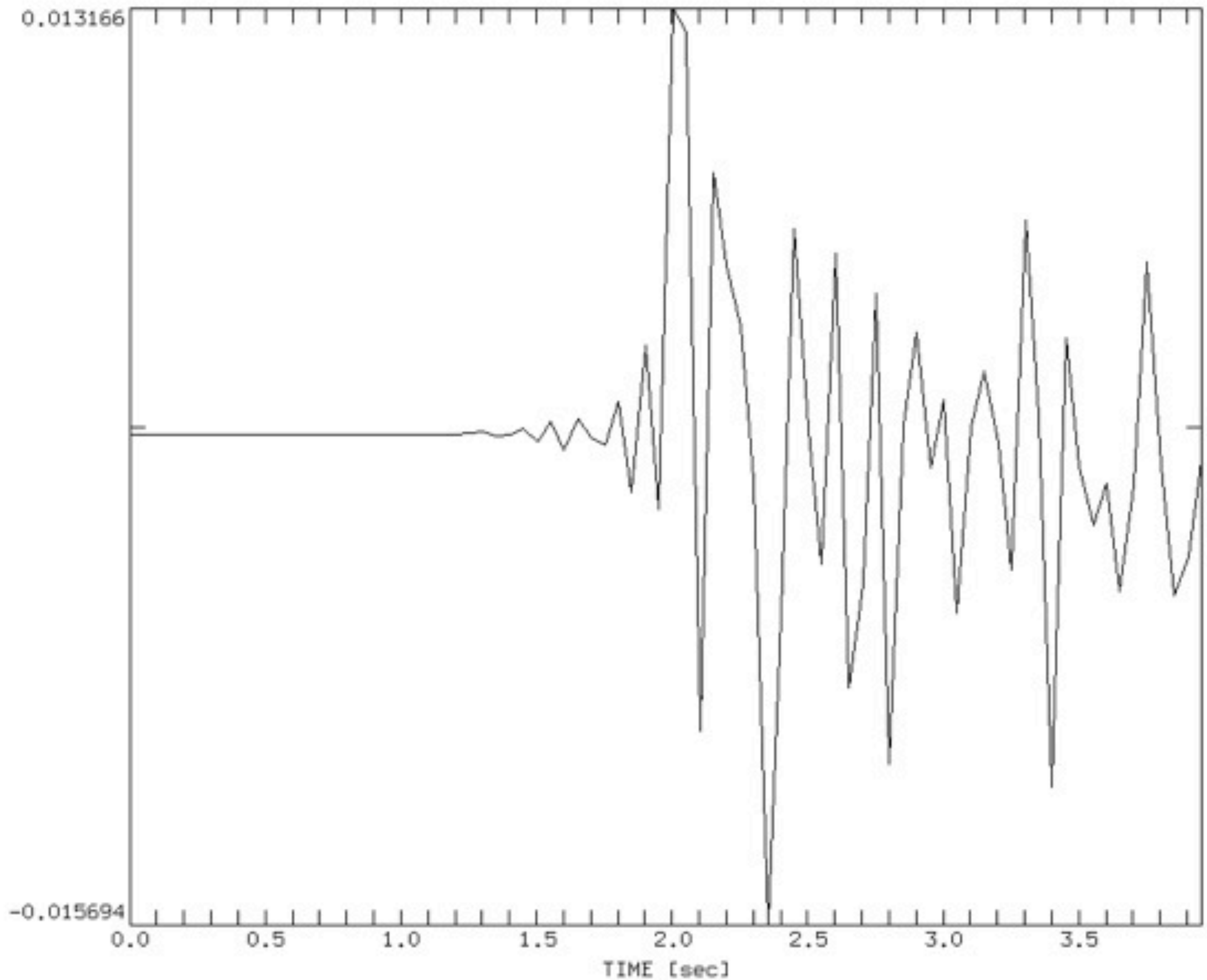
Problem: Two-Sided IR

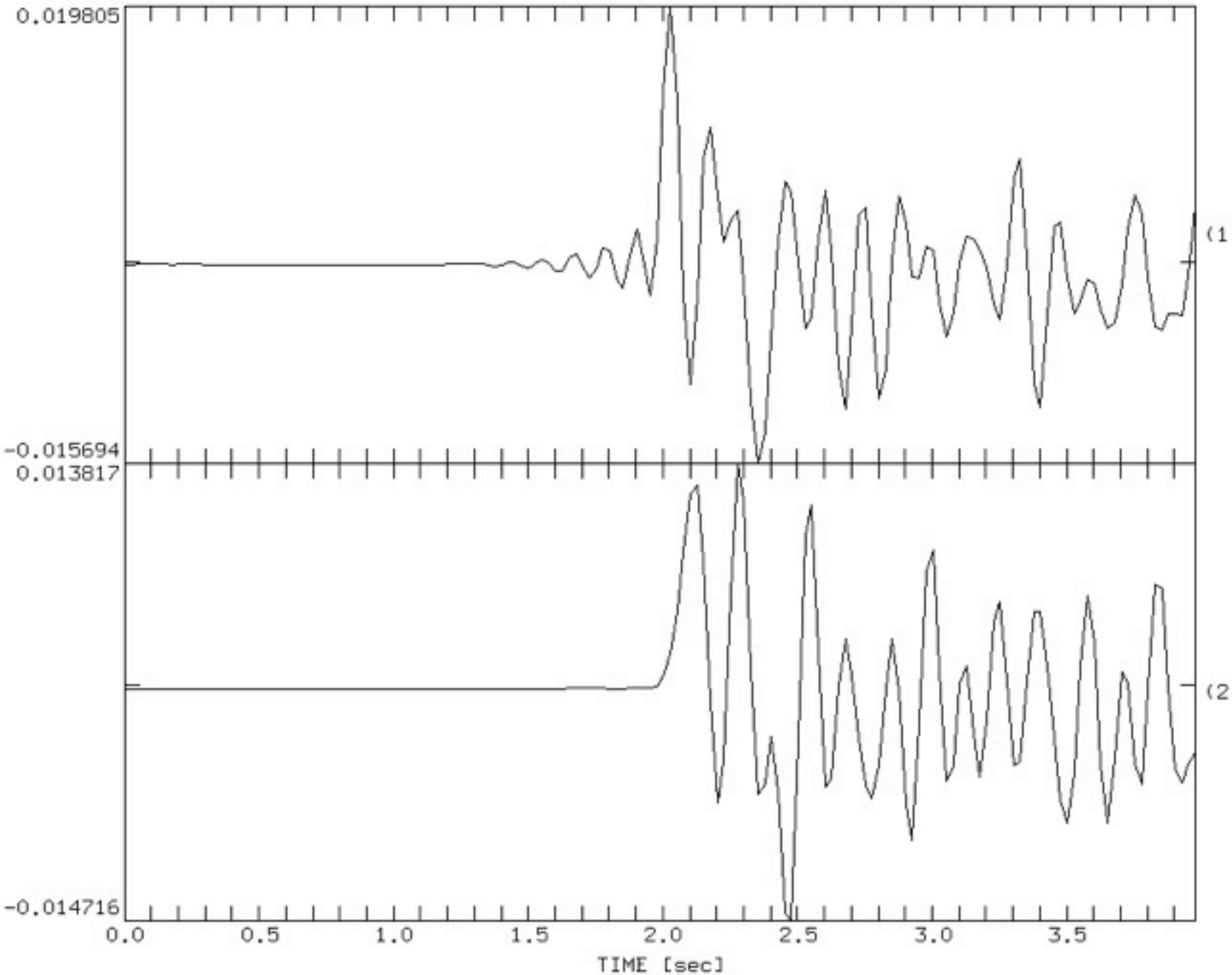
Cure: Change IR into Minimum Phase

Methods:

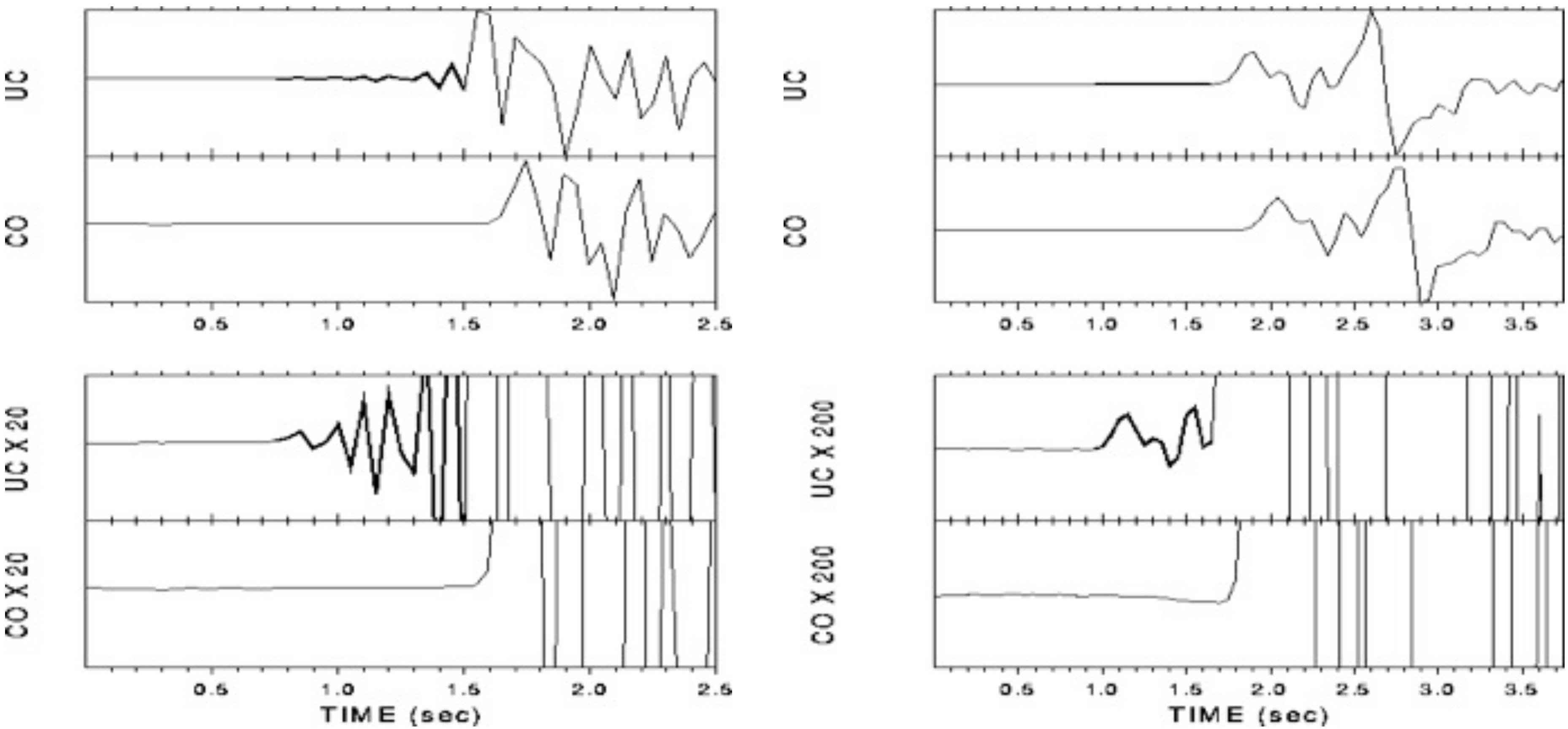
1) Add phase of Minimum Phase Filter to trace spectrum

2) Recursive Filtering of time inverted trace

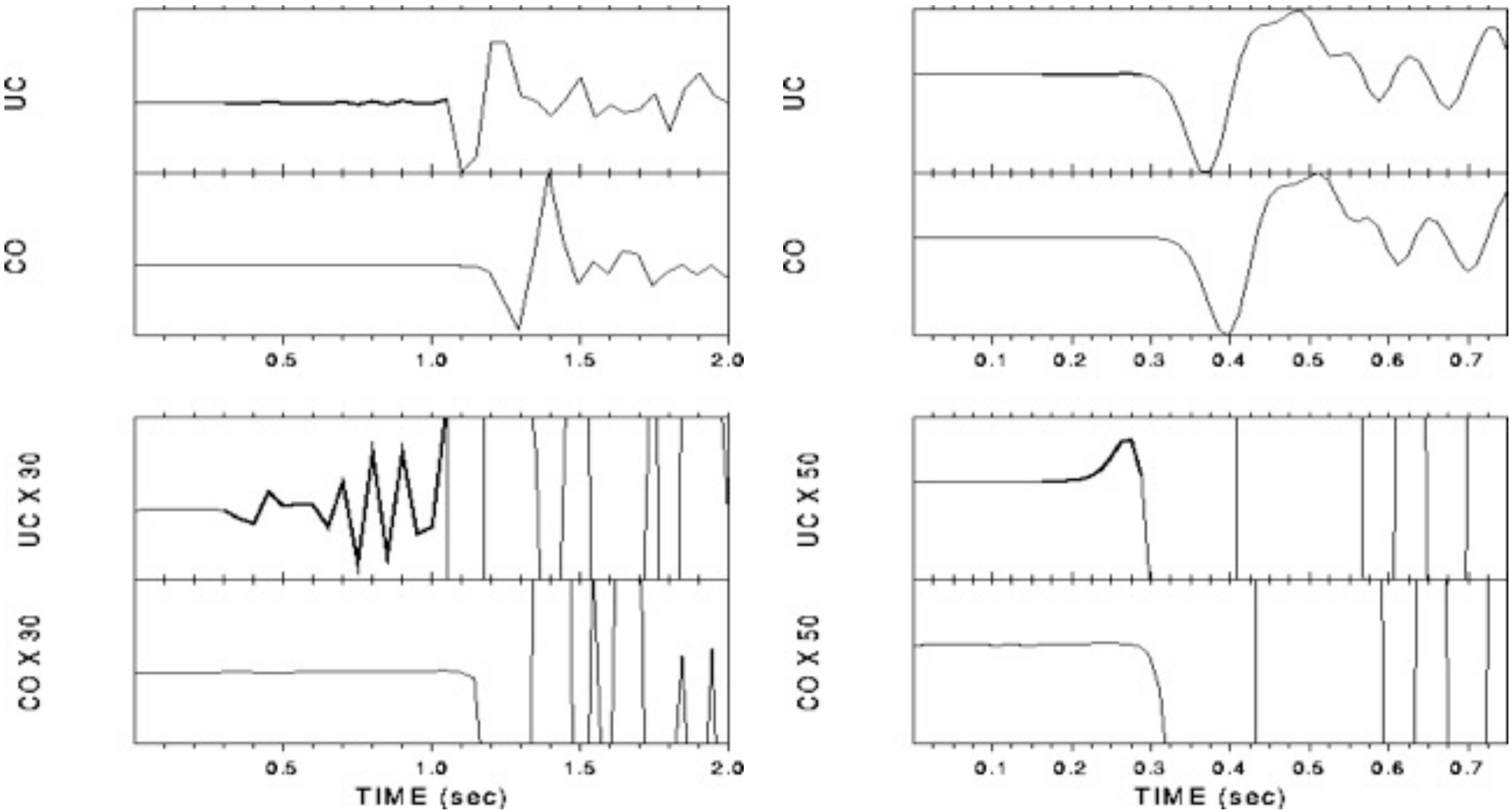

0.013166
-0.015694
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
TIME [sec]



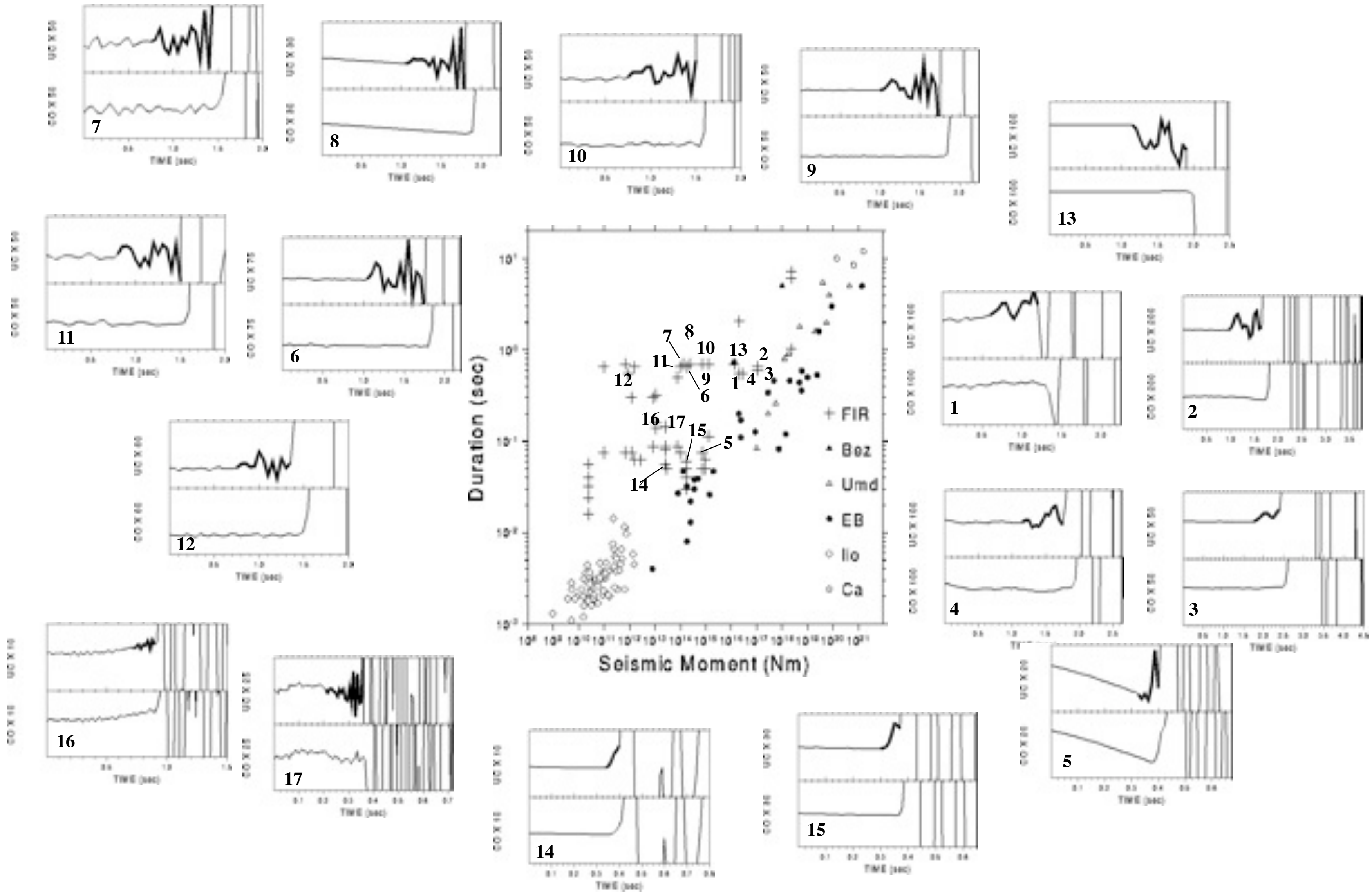
0.019805
-0.015694
0.013817
-0.014716
(1)
(2)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
TIME [sec]


UC
CO
0.5 1.0 1.5 2.0 2.5
UC X 20
CO X 20
0.5 1.0 1.5 2.0 2.5
TIME (sec)
UC
CO
0.5 1.0 1.5 2.0 2.5 3.0 3.5
UC X 200
CO X 200
0.5 1.0 1.5 2.0 2.5 3.0 3.5
TIME (sec)

a) b)


UC
CO
UC X 30
CO X 30
0.5
1.0
1.5
2.0
TIME (sec)
UC
CO
UC X 50
CO X 50
0.1
0.2
0.3
0.4
0.5
0.6
0.7
TIME (sec)

a) b)


Duration (sec)
Seismic Moment (Nm)
FIR
Bez
Umd
EB
Ilo
Ca
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
TIME (sec)

Conclusions

FIR filter generated precursory artefacts:

- **can become impossible to be identified visually**
- **can have similar scaling properties as nucleation phases**

Zero - phase FIR filters in general

- **affect the determination of all onset properties (onset times, onset polar ities)**

Consequence

For the interpretation of onset properties (onset times, onset po larities, nucleation phases, etc.) the acausal response of the zero-phase FIR filter has to be removed

but not

for waveform analysis.